QoS-based Resource Allocation in Dynamic Real-Time Systems

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***Abstract*— Dynamic real-time systems require adaptive resource management to accommodate varying processing needs. We address the problem of resource management with a single resources for soft real-time systems (no hard deadline requirements) consisting of tasks that have discrete QoS (Quality of Service) settings that correspond to varying resource usage and varying utility (benefit to the end user). Given an amount of available resource, the problem is to provide on-line control of the tasks' QoS settings so as to optimize the overall utility. Since the complexity of the problem precludes optimal solutions, we present a discrete control theory that can be implemented with a heuristic control algorithm with the following properties. (1) It has low run-time complexity, making it suitable for an on-line system. (2) It makes incremental adjustments to QoS settings as available resources change, avoiding the calculation time and instability that would be incurred by recalculating all the QoS settings. (3) Differences between actual utility and optimal do not accumulate over time, so there is no long-term degradation in performance. (4) The lower bound on actual utility can be calculated dynamically based on current system conditions, and an absolute lower bound can be calculated statically in advance. (5) It uses feedback mechanisms to respond to actual resource availability, allowing all resources to be used and tolerating misspecification of task resource requirements.**

I. Introduction

Traditional approaches to the design and development of real-time systems use worst-case execution times of tasks and provably correct analysis techniques to guarantee that critical system tasks will always complete prior to their deadlines. The most common approach is *rate monotonic analysis* [14] which uses bounds on processor utilization to guarantee that real-time periodic tasks always meet their deadlines. These traditional (and often pessimistic) approaches to the design and development of real-time systems may result in systems that are *under-utilized* for much of the operational time the system, resulting in poor quality of service. A number of factors may lead to this poor performance, including overly pessimistic worst-case execution times, environmental conditions, infrequent *critical instants* [14], inaccurate resource profiles, etc.

The discrete controller that we describe controls system resource usage by modifying quality of service parameters so that the overall system benefit is maximized. When the system resource being controlled is processor utilization, our approach is directly applicable to real-time systems since total processor utilization provides a schedulability test, i.e., a guarantee that real-time tasks will meet their deadlines whenever the processor utilization is less than or equal to some value. Since the discrete controller cannot guarantee that the resource usage is bounded, our approach is not applicable to hard-real systems where deadline can never be missed. However, our approach has many advantages in soft real-time systems where deadlines may be missed occasionally.

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A. Related Work

The problem of allocating resources to real-time applications has been studied in literature from different angles. Several authors have addressed resource allocation for real-time systems with QoS constraints. Burns et al [5] define a task model where each task is given a preference at run-time and a set of service alternatives that can be used to complete the task. QoS-based dynamic resource management tries to guarantee the feasibility of real-time systems as well as let the system produce maximum benefit, but does not explicitly address problems that arise from environmental run-time variations. Humphrey et. al. [9] propose the DQM architecture, which allows choosing operating levels to control the resource requirements for an application. The DQM monitors the performance of applications and interacts with the operating system in order to change the application to a lower operating level. The DQM is intended for use with soft realtime multimedia applications, so adaptation may occur over several periods. However, DQM uses a worst case execution time analysis to determine application resource usage. DQM does not provide a mathematical optimization model, and does not guarantee that optimal, or even near-optimal, choices have been made. In the QuO [10], [12], [23], [15] framework, applications adjust their own service levels to improve performance and react to the environment on their own accord, so there is no way to globally optimize the set of choices made for all applications. Nor is QuO able to guarantee real-time deadlines. In Q-RAM [21], [21], [19], [20], [13], an algorithmic approach is developed to find an allocation of tasks to resources such that the system can satisfy some quality of service requirements as well as produce maximum benefit. The authors do not explicitly consider employing Q-RAM for QoS optimization in dynamic environments.

The application of control-theoretic methods to the design of real-time systems has recently met with considerable success. Common challenges in real-time system design such as nonlinear and stochastic plant models, effector limitations, unknown disturbances, and noisy sensor data identified in [8] indicate a strong connection with control theory and applications. A series of works [25], [16], [24], [17], [2] address performance specifications, mathematical modeling, controller design, and performance analysis for scheduling problems in soft real-time systems. Their feedback control architecture is realized in middleware called ControlWare and its effectiveness for quality of service control is demonstrated in a web server environment. Limitations of linear systems and control methods are discussed in [1] with remedies presented that draw from scheduling and queueing theory. Related studies involving a variety of real-time system applications, performance objectives, mathematical modeling approaches, and feedback control architectures can be found in [3], [6], [7], [11], [18], [26], [22] A common thread through much of this work is that quality of service attributes of soft real-time tasks are adjusted via feedback control based on on-line system measurements. The underlying performance objective is then to optimize an aggregate quality of service metric while adhering to resource constraints and coping with an uncertain dynamic environment.

B. Contribution

This paper addresses the problem of resource management with a single resources for soft real-time systems (no hard deadline requirements) consisting of tasks that have discrete QoS (Quality of Service) settings that correspond to varying resource usage and varying benefit to the end user. Given an amount of available resource, the problem is to provide on-line control of the tasks' QoS settings so as to optimize the overall system benefit. Since the complexity of the problem precludes optimal solutions, we present a generic, and an improved heuristic algorithm with the following properties.

The low run-time complexity make them suitable to be employed as an online algorithm in dynamically changing environments. Unlike Q-RAM, our approach makes incremental adjustments to QoS settings as available resources change, avoiding the calculation time and instability that would be incurred by recalculating all the QoS settings. Differences between actual utility and optimal do not accumulate over time, so there is no long-term degradation in performance. We derive lower bounds on actual utility. They can be calculated dynamically based on current system conditions, and an absolute lower bound can be calculated statically in advance. We uses feedback mechanisms to respond to actual resource availability, allowing all resources to be used and tolerating misspecification of task resource requirements.

The remainder of this paper is organized as follows. Section II presents the system model and the model description. Section III first proposes a generic heuristic algorithm and derives theoretical properties for it. In addition, a improved heuristic is presented. Section IV describes how the algorithms can be implemented. Section V presents a performance comparison of the proposed algorithms.

II. System Model and Problem Description

A. System Architecture

Real-time tasks may be periodic or aperiodic and have associated deadlines. The tasks are classified as hard or soft. *Hard real-time tasks* must meet their deadlines. *Soft real-time tasks* will optimally meet their deadlines but need not if sufficient resources are unavailable. Soft real-time tasks can have Quality of Service (QoS) parameters, with an associated *utility*; that is, a function based on QoS settings that gives the benefit to the larger system of running the task at those settings.

The Resource Manager (RM) is middleware that provides resource management service to a dynamic real-time system. Available resources include hosts and other resources such as memory, network, disk, and so forth. The allocation manager (AM) component is responsible for making appropriate allocations of resources to tasks. Within a given host there is a scheduler and a QoS Manager (QM). The QM is responsible for dynamically optimizing the QoS settings of the soft real-time tasks with respect to utility.

In this paper we focus on the QoS Manager and hence on soft real-time tasks running on a single host. We plan to extend our approach to utility optimization across hosts, to the scheduling of hard real-time tasks, and to Allocation Manager development. The task of the QM is to optimize

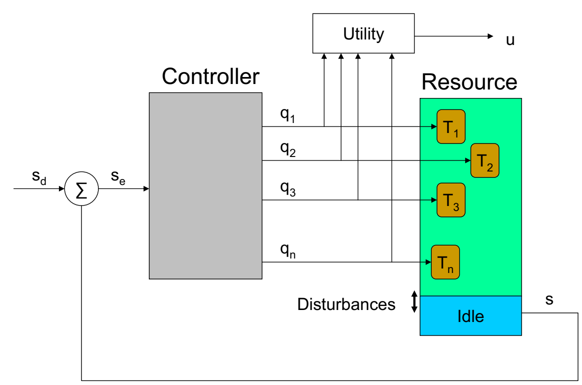

Fig. 1. Feedback Approach.

the QoS settings with respect to utility based on dynamically varying resource availability. This is accomplished via closed-loop feedback control as shown in Fig. 1. Herein, the parameter $s_d > 0$ represents the desired slack of resources. There are various ways to choose this user-defined parameter. For example, if we associate deadlines with the real-time applications and schedule the tasks according to the Rate Monotonic Scheduling (RMS) algorithm, we could use $s_d \approx 30\%$.

The applications expose the QoS attribute settings as a knob the controller can turn to change both the quality of service attributes q_i and the resource usage of the real-time tasks. In order to calculate the resource usage for a selected QoS setting, the controller makes use of resource profiles that allow to specify the estimated resource usage as a function of QoS attribute settings. Each real-time application is characterized by a utility function that expresses the user-perceived benefit from running the application at a certain QoS attribute setting. The overall benefit from running all that tasks allocated to a given host at given QoS attribute settings is measured by some aggregate of the individual (per-task) utility functions. The controller aims to always run the tasks in states for which the total utility is maximized. The host controller also monitors the actual current availability of the resource s. Disturbances in the resource utilization due to changes in the environment and dynamically arriving or terminating tasks, as well as inaccurate resource profiles will generally lead to an error in the predicted resource usage of the tasks.

This paper only addresses the QoS Manager problem, that is, the problem of determining optimal QoS settings for all soft real-time tasks. In the following we will introduce the formal task and system model.

B. Basic Model

As we have seen, the QoSM problem consist of adjusting the QoS attributes of the soft real-time applications.

Here, we assume a simple model for dynamic real-time systems. A dynamic real-time system consists of a single resource $\mathcal{R}$, available in $R \in \mathbb{R}^+$ units, and a collection of n independent periodic (soft real-time) tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$. Both the number of tasks n and the availability of the resource may vary over time. Each task T_i is a periodic activity and characterized by a period π_i. The period is not necessarily fixed and may undergo dynamic changes at runtime.

We introduce some notation used throughout the paper:

- We use two basic measures that represent time in our model: The first is the point in time as denoted by $t \in \mathbb{R}^{\geq 0}$. The second measure is the number of the k_i^{th} periodic interval of task T_i. The k_i^{th} instance of task T_i is denoted by $T_i(k_i)$.
- With each task T_i we associate a (possibly) multidimensional *Quality-of-Service* (QoS) vector $\mathbf{q_i}(k_i) \in \mathcal{Q}_i$, for interval k_i, that takes values from a finite (not necessarily numerical) set of possible choices for QoS inputs $\mathcal{Q}$. Typical examples of QoS levels include frame rate, cryptographic security, compression method, etc.
- The *utility* of a task T_i is measured by function $u_i : \mathcal{Q}_i \to \mathbb{R}$. The value $u_i(\mathbf{q_i})$ specifies the user-perceived benefit from running task T_i at QoS level $\mathbf{q_i} \in \mathcal{Q}_i$.
- The *total system utility* $u : \mathcal{Q}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_n \to \mathbb{R}$ is defined to be the sum of the task utilities, i.e. $u(\mathbf{q_1}, \mathbf{q_2}, \ldots, \mathbf{q_n}) = \sum_{i=1}^{n} u_i(\mathbf{q_i})$.
- A *resource profile* $\rho_i^{tot} : \mathcal{Q}_i \to \mathbb{N}$. The value $\rho_i^{tot}(\mathbf{q_i})$ specifies the total amount of resources necessary to achieve QoS level $\mathbf{q_i} \in \mathbb{Q}_i$. Each task T_i has a minimal resource requirement which is denoted by ρ_i^{min}. $\rho_i^{add}(\mathbf{q_i})$ denotes the amount of the resource in addition to ρ_i^{min} that is required to achieve QoS level $\mathbf{q_i}$. We assume there exists a QoS setting $\mathbf{q_{i,0}}$ such that $\rho_i^{add}(\mathbf{q_{i,0}}) = 0$.

By $r_i(k_i)$, we denote the actual amount of resources consumed by T_i as measured (and monitored) by the Resource Manager during the k_i^{th} interval.

Let t denote a point in time. We define the *actual resource slack* of the system as

$$s(t) = R - \sum_{i=1}^{n} r_i \left(\left\lfloor \frac{t}{\pi_i} \right\rfloor \right),$$

where R specifies the current maximum availability of resource $\mathcal{R}$. The error between the actual slack and the desired slack is denoted by s_e.

In a dynamic environment tasks may enter, leave, or the required resource requirements change due to the environment, or imprecise knowledge of the profiles. In our approach, the resource manager will monitor the $s_e(t)$ during intervals k_i, and make adjustments in the tasks QoS settings for the next interval $k_i + 1$ in $\mathbf{q_i}(k_{i+1})$ such that some measure of the aggregate system utility is optimized.

Let ρ describe some amount of resource availability. The utility profile $u_i^* : \mathbb{R}^{\geq 0} \to \mathbb{R}$ of a task T_i is defined to be the solution to the following optimization problem:

$$u_i^*(\rho) = \max_{\mathbf{q} \in \mathcal{Q}_i} [u_i(\mathbf{q}))]$$

such that $\rho_i^{tot}(\mathbf{q}) \leq \rho + \rho_i^{min}$.

The value $u_i^*(\rho)$ is the maximum benefit achieved by allocating $\rho \in \mathbb{R}^{\geq 0}$ amount of resource to task T_i beyond ρ_i^{min}. Clearly, we have that $u_i^*(\rho) = 0$ for $\rho = 0$. We define the $\mathbf{q_i^*}(\rho)$ as the $\mathbf{q_i}$ that maximize $u_i^*(\rho)$. Note that Rajkumar *et al.* have proposed a solution to this problem [19].

For task T_i assume that ρ can be set to any discrete value in the sequence $\langle \rho_{i,m} \rangle$. Without loss of generality, we assume that $\Delta\rho_{i,m} = \rho_{i,m+1} - \rho_{i,m} > 0$. Let

$$\Delta\rho_{min} = \min_{i=1}^{n} \left[\min_{m=0,1,\ldots} [\Delta\rho_{i,m}] \right]$$

and

$$\Delta\rho_{max} = \max_{i=1}^{n} \left[\max_{m=0,1,\ldots} [\Delta\rho_{i,m}] \right].$$

Moreover, let

$$\Delta u_{i,m}^* = u_i^*(\rho_{i,m+1}) - u_i^*(\rho_{i,m})$$

and

$$\mathbf{d}u_{i,m}^* = \Delta u_{i,m}^* / \Delta\rho_{i,m}.$$

We assume that the utility profiles $u_i^*(\rho_{i,m})$ are increasing and concave, that is, $\Delta u_{i,m}^* > 0$, and $\mathbf{d}u_{i,m+1}^* - \mathbf{d}u_{i,m}^* < 0$. Note that the utility profiles can be calculated off-line and stored in tables.

C. Optimization Problem 1 (OP1)

Now, we can define the DQRAM optimization problem. In the following, the desired actual resource availability is described by R_d, which represents the available amount of resources beyond the minimum resource requirements.

DQRAM Optimization Problem: Given utility profiles $u_i^*,\ i = 1, 2, \ldots, n$. Determine QoS settings $\mathbf{q_i}$ for all tasks T_i such that $u^*(\rho)$ is maximized subject to

$$\sum_{i=1}^{n} \rho_i^{add}(\mathbf{q_i}(k_i)) \leq R_d.$$

The above formulation of the optimization problem is equivalent to the following problem formulation which we will use in the remainder of this paper.

Optimization Problem 1 (OP1): Given utility profiles $u_i^*,\ i = 1, 2, \ldots, n$. For each $i = 1, 2, \ldots, n$ determine an index $m[i]$ for the sequence $\langle \rho_{i,m} \rangle$, such that the sum

$$U(\mathbf{m}) = \sum_{i=1}^{n} u_i^*(\rho_{i,m[i]})$$

is maximized subject to constraints

$$\rho(\mathbf{m}) := \sum_{i=1}^{n} \rho_{i,m[i]}^{add} \leq R_d,$$

where $\mathbf{m}$ denotes the sequence of indices $m[i],\ \ i = 1, 2 \ldots, n$. Note that the problem OP1 is, in general, NP-complete. This can be shown straightforwardly by reduction from the subset sum problem. But as we will see, there are cases for which the problem can be solved in polynomial time. Also, we will present a good heuristic for the general case.

III. Algorithmic Approaches

In this section we present and analyze an algorithm for problem OP1. The algorithm is depicted in Figure III and is referred to as A1. Given indices $\mathbf{m}$. Let

$$\mathbf{d_{max}} = \max_{i=1}^{n} \left[\mathbf{d}u_{i,m[i]}^* \right].$$

Property 3.1 (P1): Suppose $\mathbf{m}$ is selected such that the following equation $\mathbf{d}u_{i,m[i]-1}^* \geq \mathbf{d_{max}},\ i = 1, 2, \ldots, n$, is satisfied. That is, the slope of u^* before each index $m[i]$ is greater than or equal to the slopes after. Then $\mathbf{m}$ is said to satisfy $P1$.

The first lemma establishes the complexity of the algorithm A1.

Lemma 3.1: The complexity of Algorithm A1 is given by $\mathcal{O}\left(\frac{R_d}{\Delta\rho_{min}} \cdot \log(n)\right)$.

Proof: The values of $\mathbf{d}u_{i,m[i]}^*$ can be maintained in a heap. Then the complexity of the first line in the while loop is simply the complexity of maintaining the heap, or $\mathcal{O}(\log(n))$. Every time the loop is executed, r is incremented by at least $\Delta\rho_{min}$, so the number of times through the loop is bounded by $\left\lceil \frac{R}{\Delta\rho_{min}} \right\rceil$ ∎

Lemma 3.2: Algorithm A1 terminates with an $\mathbf{m}$ that satisfies P1.

Proof: Since the algorithm adds increments of $\Delta u_{i,m[i]}^*$ to U in order of decreasing $\mathbf{d}u_{i,m[i]}^*$, and for a given task T_i, the values of $\mathbf{d}u_{i,m[i]}^*$ are monotonically decreasing, when A1 terminates, $\mathbf{d}u_{i,m[i]+k}^* < \mathbf{d}u_{i,m[i]}^*$ for all $1 \leq i \leq n$ and $k > 0$. Therefore, $\mathbf{m}$ must satisfy P1. ∎

Algorithm 1 QoS Optimization (A1)

Input:	R_d	the maximum amount of resources to be allocated
Output:	$\mathbf{m}$	the index of each utility profile to be used
	U	the total utility achieved by this assignment

set $r = 0$;
set $U = 0$;
set $m[i] = 0$, **for all** i;
while $r \leq R_d$ **do**
  determine j **such that** $\mathbf{d}u_{j,m[j]}^* \geq \mathbf{d}u_{i,m[i]}^*$ **for all** i;
  set $r = r + \Delta\rho_{j,m[j]}$;
  set $U = U + \Delta u_{j,m[j]}^*$;
  set $m[j] + +$;
end while
/** went one increment too far **/
set $r = r - \Delta\rho_{j,m[j]}$;
set $U = U - \Delta u_{j,m[j]}^*$;
set $m[j] - -$;

Theorem 3.1: Given the sequence of indices $\mathbf{m}$ which satisfies P1 and $R_d = \rho(\mathbf{m})$, then these indices also solve OP1.

Proof: The proof is by contradiction. Suppose there exists a $\mathbf{m}' \neq \mathbf{m}$ such that $U(\mathbf{m}') > U(\mathbf{m})$ and $\rho(\mathbf{m}') \leq \rho(\mathbf{m})$. Clearly not all $m'[i]$ can be less than their corresponding $m[i]$'s. Since $\Delta u_{i,m[i]}^* > 0$ for all i and $\mathbf{m}$, this would only decrease $U(\mathbf{m}')$ from the original value of $U(\mathbf{m})$. Let $\mathcal{G}$ be the set of i where $m'[i] > m[i]$ and $\mathcal{L}$ be the set of i where $m'[i] < m[i]$. Then

$$\begin{aligned} U(\mathbf{m}') &= U(\mathbf{m}) + \sum_{i\in\mathcal{G}} \sum_{j=m[i]+1}^{m'[i]} \mathbf{d}u_{i,j}^* \Delta\rho_{i,j} \\ &\quad - \sum_{i\in\mathcal{L}} \sum_{j=m'[i]+1}^{m[i]} \mathbf{d}u_{i,j}^* \Delta\rho_{i,j} \\ &\leq U(\mathbf{m}) + d \left(\sum_{i\in\mathcal{G}} \sum_{j=m[i]+1}^{m'[i]} \Delta\rho_{i,j} \right. \\ &\quad \left. - \sum_{i\in\mathcal{L}} \sum_{j=m'[i]+1}^{m[i]} \Delta\rho_{i,j} \right) \qquad (1) \\ &= U(\mathbf{m}) + \mathbf{d_{max}} \left(\rho(\mathbf{m}) - \rho(\mathbf{m}') \right), \end{aligned}$$

where $\mathbf{d_{max}} > 0$. Since the right term of the sum in (1) must be non-negative, we have that $U(\mathbf{m}') \leq U(\mathbf{m})$, hence the contradiction ∎

Corollary 3.1: Let $\mathbf{m}$ be the sequence of indices which result from executing A1 with the rescource constraint R_d.

If $\rho(\mathbf{m}) = R_d$ then m solves OP1.

Proof: The result follows directly from the application of Theorem 3.1 and Lemma 3.2. ∎

Corollary 3.1 basically derives a sufficiency condition for an optimal solution to OP1, i.e., a criterion to see when the output of A1 solves OP1.

Theorem 3.2: Let the discretation of the ρ_i's be equidistant, that is, $\Delta\rho_{min} = \Delta\rho_{max}$. Then algorithm A1 will result in a solution of OP1 for all constraints $R_d > 0$.

Proof: Let $\mathbf{m}$ be the output of A1 for input R_d. Let $R'_d = \rho(\mathbf{m})$. if $R_d = R'_d$ then by Corollary 1 $\mathbf{m}$ solves OP1. Now suppose $R_d > R'_d$. It is clear that $R'_d + \rho_{j,m[j]+1} > R_d$ for all $1 \leq j \leq n$ since all $\rho_{j,m[j]+1}$ are equal. Hence, any indices $\mathbf{m}$ that satisfy R_d must also satisfy constraint R'_d, and again by Corollary 1, the output of A1 satisfies OP1. ∎

Now we examine the case where the ρ_i's are not equidistant. There are cases where A1 will no longer return an $\mathbf{m}$ that satisfies $OP1$. Before examining these situations, notice that Lemma 3.1 did not depend on the equidistant property. Therefore, the output $\mathbf{m}$ of $A1$ will solve $OP1$ whenever its input constraint $R_d = \rho(\mathbf{m})$. Clearly the number of distinct indices $\mathbf{m}$ that A1 produces is equal to the sum of the size of the sequences $\langle\rho_{i,m}\rangle$, or

$$N = \sum_{i=1}^{n} |\langle\rho_{i,m}\rangle| \, .$$

That is, there are N values of R_d that are distributed in intervals no smaller than $\Delta\rho_{min}$ and no larger than $\Delta\rho_{max}$ for which algorithm $A1$ will optimally solve $OP1$. So $A1$ may not give the correct solution to $OP1$ all the time when ρ_i's are not equidistant; however, when it does not, a small increase (decrease) in R_d will produce a correct solution. In other words any error in $A1$ will not accumulate. Further, we can bound the error by

Lemma 3.3: Let $\mathbf{m}'$ be the output of $A1$ for some constraint R'_d and $\mathbf{m}$ be the solution of $OP1$ for the same constraint. Then

$$U(\mathbf{m}) \leq U(\mathbf{m}') + \mathbf{d_{max}} \cdot p,$$

$p = \max_{j \in \mathcal{G}} \rho_{j,m'[j]}$, where $\mathcal{G} = \left\{ i | \ \mathbf{d}u^*_{i,m'[i]} = d \right\}$

Proof: Assume $\rho(\mathbf{m}') < R'_d$, then $\mathbf{m}'$ must solve OP1 for the constraint $R_d = \rho(\mathbf{m}')$. Therefore $U(\mathbf{m})$ cannot exceed $U(m')$ by more than the maximum slope of the utility profiles (or simply $\mathbf{d_{max}}$) times $R'_d - R_d$. But $R'_d - R_d$ cannot exceed p, otherwise, the algorithm would have incremented $m'[j]$. ∎

Now we examine the causes and possible solutions to problems introduced into A1 when ρ_i's are not equidistant. We identify three situations where the output of A1 may not be the solution of OP1.

Issue 1 (I1). Suppose the algorithm A1 exits with $r < R_d$, and the values for $\mathbf{d_{max}}$ have been equal for the past

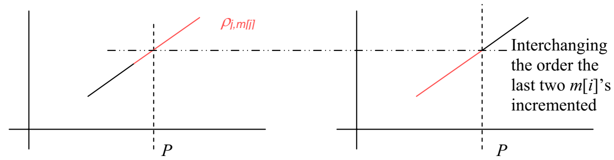

Fig. 2. Visualization for Issue 1.

$q > 1$ times through the loop. Let

$$\begin{aligned} \mathcal{J} &= \left\{ j | \ \mathbf{d}u^*_{j,m[j]-1} = \mathbf{d_{max}} \right\} \\ \mathcal{K} &= \left\{ k | \ \mathbf{d}u^*_{k,m[k]} = \mathbf{d_{max}} \right\} \end{aligned}$$

then incrementing $m[i]$ of some of the tasks $i \in \mathcal{K}$ while decrementing the $m[i]$ for tasks in $i \in \mathcal{J}$ may result in a better solution (see Figure 2).

Issue 2 (I2). The algorithm terminates when $p + \Delta\rho_{j,m[j]}$

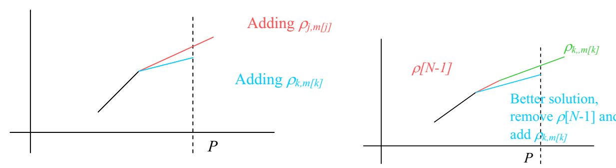

Fig. 3. Visualization for Issue 2 (left) and Issue 3 (right).

exceeds R_d. There may exist an $k \neq j$, where $p + \Delta\rho_{k,m[k]} \leq R_d$. In this case incrementing $m[k]$ will result in a better solution (see Figure 3).

Issue 3 (I3). The algorithm terminates when $p + \Delta\rho_{j,m[j]}$ exceeds R_d. Let N denote the number of times the while loop in A1 executed, and $\Delta p[i]$ and $\Delta u[i]$, $(1 \leq i < N)$ be the amounts that p and U were incremented during the i^{th} iteration of the loop. Note that $i < N$, since the last increment is removed outside the loop. There may exist an $k \neq j$ and an $1 \leq l < N$ where

$$p + \Delta\rho_{k,m[k]} - \sum_{i=1}^{l} \Delta\rho[N-i] \leq R_d,$$

and

$$\Delta u_{k,m[k]} - \sum_{i=1}^{l} \Delta u[N-i] > 0,$$

then incrementing $m[k]$ and decrementing indices corresponding to tasks that were incremented during the last n iterations of the loop, will result in an improved solution (see figure 3).

To totally resolve all three issues is in general a NP-hard problem. This is easy to see since I1 by itself is equivalent to the knapsack problem with the weights of the knapsack items equal to the $\rho_{i,j}$-values, the values of the knapsack items equal to the $u^*_{i,j}$-values, and finally, the capacity of the knapsack equal to the remaining slack R.

However, we can develop a good heuristics to take advantage of the structure of the problem that will help to reduce the affects of the three issues. First, if there are more than one task with equal $\mathbf{d}u^*_{i,m[i]}$, then in the while loop choose the one with largest $\Delta\rho_{i,m[i]}$. Next, after exiting the loop, examine each entry in the heap in order (except for the top element which caused the loop to exit) to see if it can be added, if so it is added. If the task cannot be directly added but it can be added if $\Delta\rho[N-1]$ is removed and the resulting utility is increased, then make the adjustments. Finally, we modify the algorithm for incremental evaluation. These modifications are presented in algorithms *Allocate (A2)* and *Free (A3)*.

Algorithm A2 initially follows the logic behind A1 except it initializes $r = R_0$, $U = U_0$, and $\mathbf{m} = \mathbf{m}_0$. After the first loop is exited, the heap is searched in order in the second loop. This loop looks for two situations. First, if incrementing the index $m[j]$ will increase the system utility without exceeding the constraint then it is incremented Second, if incrementing $m[j]$ exceeds the constraint, the loop makes one more attempt to see if incrementing $m[j]$ will help. It checks if incrementing $m[j]$ in conjunction with decrementing the index of the last task added by the first loop will meet the resource constraint. If so, it further checks if this will increase the system utility. If it does it performs the modifications to the indices.

This amazingly simple heuristic modification addresses all three issues. When tasks have equal $\mathbf{d}u^*_{j,m[j]}$ they will be added in decreasing order of $\Delta\rho_{j,m[j]}$. When the first loop is exited, all the remaining tasks T_i with $\mathbf{d}u^*_{i,m[i]} = \mathbf{d}u^*_{j,m[j]}$ will be searched in decreasing order of $\Delta\rho_{j,m[j]}$ and added if there is room. This addresses issues I1 by using the largest first heuristic, which has been shown to give good results [4]. Once all the tasks with equal $\mathbf{d}u^*_{i,m[i]}$ have been searched, the second loop continues until the end of the heap. It will increment $m[i]$ for tasks with slopes less than $\mathbf{d}u^*_{j,m[j]}$ that meet the constraint. This clearly addresses issue I2 in a greedy fashion. If $m[i]$ cannot be incremented without exceeding the constraint, but can be if $m[last]$ is decremented, then these adjustments are made only if an improvement in utility is achieved. Since the tasks are examined in the order they appear in the heap, then I3 is also addressed in a greedy fashion. For efficiency sake, we examine only the case where one previously incremented task is adjusted. This is a good rule as long as $\Delta\rho_{max}$ and $\Delta\rho_{min}$ are close in value.

The incremental nature of the A2 and A3 algorithms are evident. However, A2 must begin with an $\mathbf{m}_0$ that satisfies P1. This is easily done by recording the amount of resource allocated during the second loop. This amount of resource is called a *loan*. If A2 returns with a loan, we simply call A3 to free the loan first and then call A2. The resulting $\mathbf{m}$ from A3 will satisfy P1, since allocations are freed in exactly the opposite order that they are allocated in.

Lemma 3.4: The complexity of Algorithm A2 is given by $\mathcal{O}\left(\left[\left(\frac{R_d - R_0}{\Delta\rho_{min}}\right) + n\right]\log(n)\right)$ and A3 is given by $\mathcal{O}\left(\left(\frac{R_d - R_0}{\Delta\rho_{min}}\right)\log(n)\right)$

Algorithm 2 Allocate (A2)

Input:	R_d	the maximum amount of resources to be allocated
	R_0	the current resource allocation, it is assumed $R_0 < R_d$
	U_0	the current system utility
	$\mathbf{m}_0$	the current index of each utility profile, it is assumed that $\mathbf{m}_0$ satisfies P1
Output:	$\mathbf{m}$	the index of each utility profile to be used
	U	the total utility achieved by this assignment
	$loan$	the amount of resource on loan

set $r = R_0$;
set $U = U_0$;
set $m[i] = m_0[i]$, **for all** i;
set $j = 0$;
while $r \le R_d$ **do**
  set $last = j$;
  set $j =$ **index of the top tasks in** $\mathcal{H}_{max}$;
  set $r = r + \Delta\rho_{j,m[j]}$;
  set $U = U + \Delta u^*_{j,m[j]}$;
  set $m[j]++$;
  add task j **back into** $\mathcal{H}_{max}$;
end while
/** went one increment too far **/
set $r = r - \Delta\rho_{j,m[j]}$;
set $U = U - \Delta u^*_{j,m[j]}$;
set $m[j]--$;
/** see if we can fit a task that does not have the greatest derivative **/
remove top element from heap $\mathcal{H}_{max}$;
set $loan = 0$;
while $r < R_d$ **and** $\mathcal{H}_{max}$ **is not empty do**
  set $j =$ **index of the top task in** $\mathcal{H}_{max}$;
  if $r = r + \Delta\rho_{j,m[j]} < R_d$ **then**
    set $loan = loan + \Delta\rho_{j,m[j]}$;
    set $r = r + \Delta\rho_{j,m[j]}$;
    set $U = U + \Delta u^*_{j,m[j]}$;
    set $m[j]++$;
  else if $p + \Delta\rho_{j,m[j]} - \Delta\rho_{last,m[last]} < R_d$ **and** $\Delta u^*_{k,m[k]} - \Delta u^*_{last,m[last]} > 0$ **then**
    set $loan = loan + \Delta\rho_{j,m[j]} - \Delta\rho_{last,m[last]}$;
    set $r = r + \Delta\rho_{j,m[j]} - \Delta\rho_{last,m[last]}$;
    set $U = U + \Delta u^*_{j,m[j]} - \Delta u^*_{last,m[last]}$;
    set $m[j]++$;
    set $m[last]--$;
  end if
end while

Algorithm 3 Free (A(3)

Input:	R	the maximum amount of resources to be allocated
	R_0	the current resource allocation, it is assumed $R_0 < R_d$
	U_0	the current system utility
	$\mathbf{m}_0$	the current index of each utility profile, it is assumed that $\mathbf{m}_0$ satisfies P1
Output:	$\mathbf{m}$	the index of each utility profile to be used
	U	the total utility achieved by this assignment
	$loan$	the amount of resource on loan

set $r = R_0$;
set $U = U_0$;
set $m[i] = m_0[i]$, **for all** i;
while $r > R$ **do**
  set $last = j$;
  set $j =$ **index of the top tasks in** $\mathcal{H}_{min}$;
  set $r = r - \Delta\rho_{j,m[j]}$;
  set $U = U - \Delta u^*_{j,m[j]}$;
  set $m[j] + +$;
  add task j **back into** $\mathcal{H}_{min}$;
end while

Proof: This is a direct result of Lemma 3.1 and the observation that the second loop in A2 is executed exactly n times. ■

Theorem 3.3: If Algorithm A2 terminates with $loan = 0$ and $R_d = \rho(\mathbf{m})$, then $\mathbf{m}$ solves OP1.

Proof: Since $loan = 0$, then the second loop did not change $\mathbf{m}$. Under this condition, $\mathbf{m}$ is calculated in the same way as in Algorithm A1. Hence, by Theorem 3.1, $\mathbf{m}$ solves OP1. ■

Since we free the loan before calling A2, any errors not resolved by the second loop cannot accumulate. Therefore, like A1, A2 will return exact solutions to OP1 for at least N values of R_d and these values occur at intervals no farther than $\Delta\rho_{max}$ apart. Further, it is obvious that any allocation made in the second loop of A2 only improves the system utility. So the output of A2 is equal or better than A1. Finally, if $R_d - R_0 \ll R_d$, the execution time of A2 is significant faster than A1.

IV. Simulation Results

Figure 4 presents a comparison of the performance of algorithm A1 and our improved heuristic A2+A3 with Exhaustive Search. The $x-$axis represents the available amount of resource to allocate and the $y-$axis shows the total utility achieved. The results are based on a scenario with 3 tasks with randomly generated resource profiles. For each task, the utility profiles are based on 8 randomly generated resource allocation choices. Figure 4 shows that the improved heuristic A2+A3 outperforms A1 and is very close to the optimal solution.

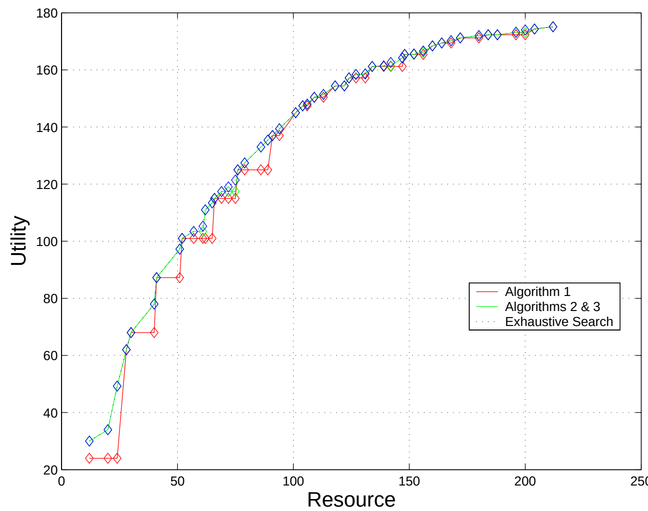

Fig. 4. Comparison of Performance of Algorithm2 A1 and A2+A3 with Exhaustive Search (optimal)

V. Implementation Aspects

The implementation of the controller is straight forward. The controller described in this paper is invoked at any of the following events: (1) Task arrival, (2) Task completion, and (3) The end of every task period.

The controller maintains a state for each task which is the current value of the index $m[i]$ for the task and the desired value for index which is denoted by $n[i]$. At the end of a period, the controller determines slack error by querying the available resource, subtracting the amount of resource reserved for future task periods by previous invocations of the controller, and finally subtracted the desired slack. This yields the excess resource to be allocated

$$s_x = s - \left(\sum_{i=1}^{n} \rho_{i,n[i]} - \rho_{i,m[i]} \right) - s_d.$$

If s_x is positive, the previous *loan* is freed, and then $R_d = R_0 + loan + s_x$ is allocated. the current values of $n[i]$ for each task are used to initialize $\mathbf{m_0}$ in allocated, and ultimately the output of allocate $\mathbf{m}$ updates these same $n[i]$'s. Similarly, if s_x is negative, $R_d = R_0 - s_x$ amount is freed the $n[i]$'s are updated.

At the beginning of each period, the task checks its $n[i]$. If $n[i] \neq m[i]$, the task updates its QoS setting to $\mathbf{q}_{n[i]}$, sets $m[i] = n[[i]$, and starts its next invocation.

When a new task T_{new} arrives, the controller temporarily assigns its index $m[new]$ to its maximum allowed value. The excess resource is calculated as follows

$$s_x = s - \left(\sum_{i=1}^{n} \rho_{i,n[i]} - \rho_{i,m[i]} \right) - s_d - \rho^{tot}_{new}\left(\mathbf{q}^*_{\mathbf{new}}\right),$$

where $\mathbf{q}^*_{\mathbf{new}}$ is the QoS level for task T_{new} for $m[new]$.

Any *loan* is freed from the original tasks and then $R_d = R_0 - (s_x - loan)$ is freed from all tasks including the new one. If free terminates with $\mathbf{m} = 0$, and R_d has not been achieved then there is not enough resources to add the tasks and it is rejected. Otherwise, the output $\mathbf{m}$ of free is assigned to the $n[i]'$s of all tasks. then all of the original tasks complete their current period and have updated their QoS settings to accomodate the new tasks, the new task is launched.

Finally, when task T_l ends, the excess resource is calculated as follows

$$s_x = s - \left(\sum_{i=1}^{n} \rho_{i,n[i]} - \rho_{i,m[i]}\right) - s_d + \rho_e^{tot}\left(\mathbf{q_e^*}\right),$$

where $\mathbf{q_e^*}$

is the QoS level for task T_e for $m[e]$. The previous *loan* is freed and then $R_d = R_0 + loan + s_x$ is allocated.

VI. Conclusions and Future Work

Extending the work of QRAM, we have presented a discrete control theory to optimize utility with respect to QoS settings on a single resource. We have shown that the theory can be implemented via an efficient and stable control algorithm that optimizes task QoS settings, within known margins, with respect to overall utility, that does not accumulate error over time, and that tolerates task resource misspecification. We have shown experimentally that the QoS settings produced by the algorithm tend to be quite good; actual utility produced is typically very close to if not identical with the optimal. In future work we intend to extend the theory in the following ways. (1) Handle multiple instances of the same resource (such as multiple hosts or multiple networks). (2) Handle multiple resource types (both hosts and networks, for instance). (3) Handle hard real-time tasks, for single and multiple instances of a resource, and for multiple resource types. (4) Handle the problem of moving tasks from one resource instance to another to ensure schedulability and optimize utility.

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